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Abstract

A complete wave profile for proforce style of breakdown waves with a current behind the shock front is discussed. The solution of the electron fluid dynamical equations in the sheath region for proforce current bearing waves conforms with the expected conditins for the values of the dynamical variables at the trailing edge of the wave. The wave profile for electric field and electron velocity, temperature, and number density are presented.

Introduction

The model is that of an infinite plane wave traveling in the positive x direction with a speed V. Assuming that the neutral particles are at rest in the laboratory frame, in the wave frame they are being swept into the wave front with a speed -V. Considering the wave front to be the origin of the wave frame, x = 0, the wave will extend to x = -∞. The wave front at x = 0 is a strong discontinuity (a shock front) which separates the neutral gas in front of the wave from the three component fluid composed of neutral particles, ions, and electrons. The structure of the wave consists of two distinct regions. First, a thin dynamical region at the wave front which is somewhat thicker than a Debye length and will be referred to as the shock layer or the sheath region. Here the electric field, electron velocity, and the electron gas temperature first attain maximum values, and then the electric field reduces to zero as the electrons slow down to the same velocity as the heavy particles. Second, a broad quasi-neutral region, where at the beginning of this region the electrons posses a high temperature, and further ionization takes place until the electrons cool below the ionizing temperature. Breakdown waves for which electron mobility is in the direction of wave propagation will be referred to as proforce waves.

A one-dimensional, steady-profile, multifluid model is employed to describe the breakdown wave propagation into a neutral medium subjected to a strong electric field. In order to describe the wave in terms of electron variables only, electron-fluid equations are decoupled from the rest of the equations. In the one-dimensional approximation, the set of electron fluid dynamical equations, composed of the equations of conservation of mass, momentum, and energy coupled with the Posisson's equation, have been applied to a wave of steady profile. The set of equations has been discussed in detail in two previous papers: (a) Fowler et al. (1984), and (b) Hemmati and Young (1995). In terms of the dimensionless variables

$$\psi = \frac{v}{V},\ \eta = \frac{E}{E_0},\ \theta = \frac{kT_e}{2e\phi},\ \nu = \frac{2e\phi}{\epsilon_0 E_0^2}n,\ \xi = \frac{eE_0}{mV^2}x,\ \alpha = \frac{2e\phi}{mV^2},\ \kappa = \frac{mV}{eE_0}K,\ \mu = \frac{\beta}{K},\ \omega = \frac{2m}{M}$$

the set of equations are

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu. \tag{1}$$

$$\frac{d}{d\xi}\left\{\nu\psi(\psi-1) + \alpha\nu\theta\right\} = -\nu\eta - \kappa\nu(\psi-1). \tag{2}$$

$$\frac{d}{d\xi}\left\{\nu\psi(\psi-1)^2 + \alpha\nu\theta(5\psi-2) + \alpha\nu\psi + \alpha\eta^2 - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}\right\} = -\omega\kappa\nu\left\{3\alpha\theta + (\psi-1)^2\right\}. \tag{3}$$

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(\psi-1). \tag{4}$$

Equations (1) through (3) are the equations of conservation of mass, momentum, and energy, respectively. With no time variation in the wave frame, Maxwell's equations in one dimension reduce to poisson's equation, equation (4), alone. The variables v, T_e, n, E, x, are electron velocity, electron temperature, electron number density, electric field (applied field plus space charge field), and position inside the sheath. In dimensionless form they are represented by ψ, θ, ν, η, and ξ. The symbol ϕ, K, β, and μ are ionization potential, elastic collision frequency, ionization frequency, and ionization rate respectively, with α and κ representing the wave parameters.

Solution of the Equations

The wave is considered to have a strong discontinuity at it's front, and in terms of nondimensional variables the shock front boundary conditions which have proven to be sucessful (Fowler et al. 1984) are

$$\nu_1 \neq 0; \quad \eta_1 = 1; \quad \theta_1 = \psi_1(1-\psi_1)/\alpha;$$

$$\psi_1 = \frac{5\left(1 + \frac{\alpha\theta_1'}{\kappa}\right) - \left\{16\alpha + \left(3 - \frac{5\alpha\theta_1'}{\kappa}\right)^2\right\}^{\frac{1}{2}}}{2}. \tag{5}$$

where ν_1, θ_1, ψ_1 are electron number density, electron temperature, and electron velocity, respectively; η_1 is the electric field, and θ_1' is the electron temperature derivative at the shock front. The conditions known to exist at the trailing edge of the sheath are

$$\eta_2 = 0; \quad \psi_2 = 1; \quad \eta_2' = 0. \tag{6}$$

To achieve solution for the set of electron fluid dynamical equations, the equations are integrated by trail and error through the sheath region, as described below. To make the integration of the set of equations through the sheath region possible, the momentum balance equation is expanded and other equations from the set are used to solve for eletron velocity derivative

$$\frac{d\psi}{d\xi} = \frac{\kappa\nu(1-\psi)(1+\mu) - \alpha\theta\kappa\mu - \alpha\psi\theta' - \eta\psi}{\psi^2 - \alpha\theta}. \tag{7}$$

This allows for the singularity inherent in the set of equations to appear in the denominator of equation (7). The movable singularity present between ψ_1 and 1 is used to integrate the set of equations by trial and error. For $0 < \psi < 1$ when $\psi^2 - \alpha\theta$ approaches zero, the electron velocity derivative will approach infinity. This represents the presence of a shock within the sheath. A shock inside the sheath is not allowed; therefore, the numerator in equation (7) has to approach zero at the same time as the denominator approaches zero. For a given wave speed α and electron number denisty ν_1, this allows choice of initial value of ψ_1 by trail and error for a given κ. After reaching the singularity, for approximately ten integration steps the denominator and the numerator in equation (7) are held constant until both change signs after passing through the singularity. The integration of the set of equations is continued until ψ approaches 1. At the trailing edge of the wave, the conditions (6), however, have to be satisfied in their entirety. If conditions (6) are not satisfied, the process has to begin with a new value of κ and ψ_1.

The ionization rate, μ, is calculated by using an equation derived by Fowler (1983). His derivation is based on free trajectory theory, where ionization due to both directed and random motion of electrons is included.

$$\mu = \mu_0 \int_{\frac{1}{\sqrt{2\theta}}}^{\infty} \sigma_i x^2 \, dx \int_{\frac{(1-\psi)}{\sqrt{2\alpha\theta}}}^{\infty} \frac{e^{-(x-u)^2} - e^{-(x+u)^2}}{u} e^{-2k\sqrt{2\alpha\theta}} \, du. \tag{8}$$

Analysis and Results

For breakdown waves with a large current behind the shock front, Poisson's equation and the initial boundary condition on electron temperature have to be modified (Hemmati and Young 1995). In dimensionless variables the modified Poisson's equation and the boundary condition on electron temperature respectively are

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(\psi - 1) + \kappa\iota, \tag{9}$$

$$\theta_1 = \frac{\psi_1(1-\psi_1)}{\alpha} + \frac{\kappa}{\nu_1}\iota, \tag{10}$$

where, $\iota = \frac{I_1}{\epsilon_0 E_0 K}$, in equations (9) and (10) is the dimensionless current, and I_1 is the current behind the shock front. Uman (1971), considering long laboratory sparks as miniature lightning, calculates the current values from electric field measurements. He reports peak currents in the order of 10^3 A for leader steps, and peak currents in the range of 10 kA - 100 kA for the lightning return stroke. These current values correspond to a dimensionless current, ι, value in the range of 0.005 and 0.5. For larger current values, the computer integration of the set of electron fluid dynamical equations becomes very difficult and time consuming. For larger current values in order to make the passage through the singularity possible, the number of integration steps near the singularity where the numerator and the denominator in equation (7) are kept constant must be increased up to twenty steps. This is the cause of the kink in the enclosed graphs. Earlier, Hemmati and Young (1995) have reported the variation of electric field and electron velocity inside the sheath for several current values. Their results meet the expected boundary conditions at the trailing edge of the sheath. A complete wave profile, for a larger current value, $\iota = 0.25$, is the subject of the present paper.

Earlier attempts in integration of the set of equations for this value of the current resulted in only the passage through the singularity. In our recent attempts, not only have we been able to pass through the singularity, but also our best result is satisfactory in meeting the conditions at the trailing edge of the sheath. For a satisfactory solution at the end of the sheath, η has to approach zero while electrons acquire the same speed as those of heavy particles ($\psi \rightarrow 1$). At the end of the sheath, the electron temperature, θ, has to remain positive. A positive electron temperature indicates electron capacity for further ionization.

Figure 1 is a graph of electric field, η, as a function of electron velocity, ψ. As expected, the beginning value of the electric field is equal to that of the applied field ($\eta = 1$), and it approches zero at the end of the sheath.

Figure 2 is a graph of electric field, η, as a function of position, ξ, inside the sheath. Comparing the sheath thickness for $\iota = 0.25$ with those of lower current values, Hemmati and Young (1995) shows that as the current value behind the shock front increases, the sheath becomes thicker.

Figures 3 and 4 are graphs of electron temperature, θ, and electron number density, ν, as a function of position inside the sheath. Our results are for $\alpha = 0.01$, which represents a wave speed value of 3×10^7 m/s, and $\kappa = 0.8033678$. For large current values the passage through the singularity requires a larger number of significant figures in variable values such as κ. The other variable val-

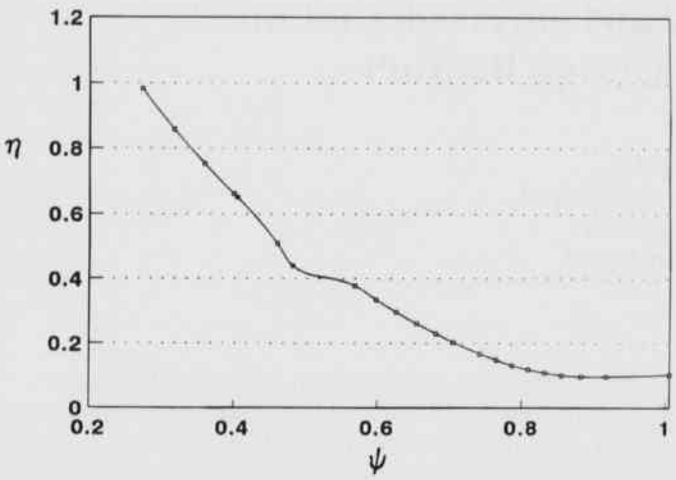

Fig. 1. Net electric field, η, as a function of electron velocity, ψ, inside the sheath.

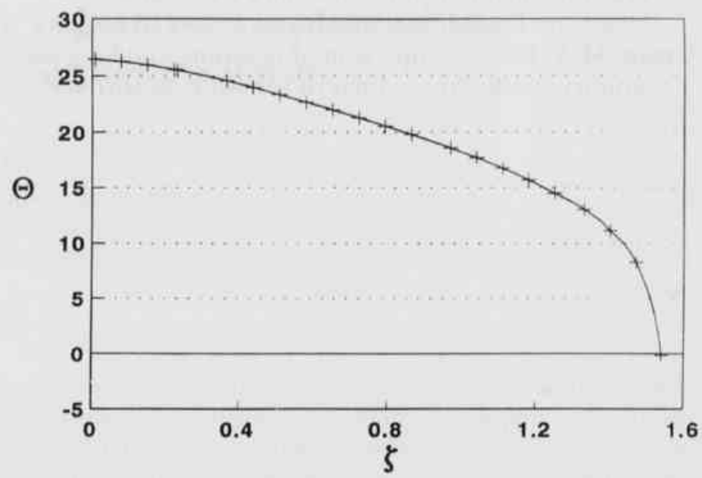

Fig. 3. Electron temperature, θ, as a function of position, ξ, inside the sheath.

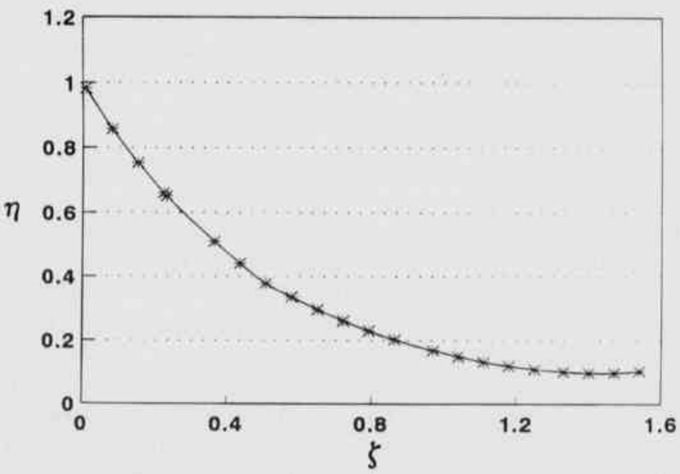

Fig. 2. Net electric field η, as a function of position, ξ, inside the sheath.

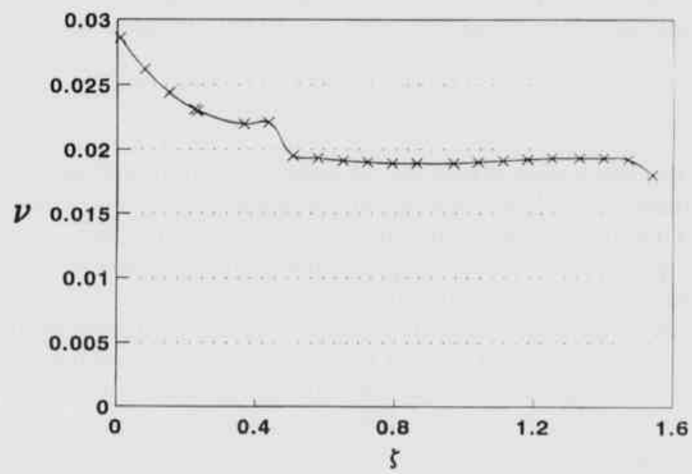

Fig. 4. Electron number density, ν, as a function of position, ξ, inside the sheath.

ues at the shock front for our successful solution are ν_1 = 0.029 and ψ_1 = 0.267. The singularity appears at ξ = 0.34.

Conclusions

For current value of ι = 0.5, the integration of the set of equations become practically impossible. This indicates the existence of a cut-off point for current values for which the solution of the electron fluid dynamical equations is possible. The existence of a cut-off point for current agrees with the reported experimental current values (Uman 1971) and is a validation of the fluid model and the set of electron fluid dynamical equations.

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Literature Cited

Fowler, R.G., M. Hemmati, R.P. Scott and **S. Parsenajadh.** 1984. Electric breakdown waves: Exact numerical solutions. Part I. Phys. Fluids. 27:1521-1526.

Fowler, R.G. 1983. A trajectory theory of ionization in strong electric fields. J. Phys. B: At. Mol. Phys. 16:4495-4510.

Hemmati, M. and **S. Young.** 1995. Proforce waves: the effect of current behind the shock front on wave structure. Proceedings of Arkansas Acad. of Sci.

Uman, M.A. 1971. Comparison of lightning and long laboratory spark. Proceedings of the IEEE. 59:457-466.